(1)

EEN 360
MODERN
CONTROL

FALL 2007
TEST 1
50 MINUTES

78/100

NDU
ECCE DEPARTMENT
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NAME:

NOTE1: OPEN BOOK, OPEN NOTES, CLOSED OLD TESTS AND SOLUTIONS
NOTE2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT.

(1) 8
(2) 25
(3) 45

1. 30 Pts. A laser printer uses a laser beam to print copy rapidly for a computer. The laser is positioned by a control input, r(t) so that we have

$$Y(s) = \frac{5(s+100)}{s^2+60s+500} R(s)$$

The input r(t) represents the desired position of the laser beam
   a) If r(t) is a unit step input, find the output y(t)
   b) What is the final value of y(t)?

a) r(t) is a unit step input $\Rightarrow r(t) = 1 \Rightarrow R(s) = 1/s$

$$Y(s) = \frac{5(s+100)}{(s)(s^2+60s+500)} \Rightarrow \text{using Laplace}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{5(s+100)}{s(s^2+60s+500)}\right)$$

$$\Rightarrow \frac{5(s+100)}{s(s^2+60s+500)} = \frac{A}{s} + \frac{Bs+D}{s^2+60s+500}$$

$$\Rightarrow A = \left.\frac{5(s+100)}{s^2+60s+500}\right|_{s=0} = \frac{500}{500} = 1$$

$$\Rightarrow A(s^2+60s+500) + Bs^2 + Ds = 5s+500$$

$$\Rightarrow As^2 + 60As + 500A + Bs^2 + Ds = 5s+500$$

$$\therefore A + B = 0 \Rightarrow \boxed{B=-1}$$

$$60A + D = 5 \Rightarrow \boxed{D=-55}$$

$$500A = 500 \Rightarrow \boxed{A=1}$$

②

a) $\dfrac{5(s+100)}{s(s^2+60s+500)} = \dfrac{1}{s} + \dfrac{-s-55}{s^2+60s+500}$

$= \dfrac{1}{s} + \left(\dfrac{s-55}{(s+30)^2-400}\right)$

$= \dfrac{1}{s} + \left(\dfrac{-s-55}{(s+30)^2-(20)^2}\right) = \dfrac{1}{s} - \left(\dfrac{s+55}{(s+30)^2+(20)^2}\right)$?

$= \dfrac{1}{s} - \left(\dfrac{s+55}{(s+30)^2+(20)^2}\right)$

$\Rightarrow y(t) = 1 - \left(\dfrac{1}{20}\left[(55-30)^2+(20)^2\right]^{1/2} e^{-30t}(\sin 20t)\right)$

$y(t) = 1 - (1.6008)\, e^{-30t} \sin 20t$

$F(s) = \dfrac{s+\alpha}{(s+a)^2+\omega^2}$

$\Rightarrow f(t) = \dfrac{1}{\omega}\left((\alpha-a)^2+\omega^2\right)^{1/2} e^{-at}\sin(\omega t)$

$\Rightarrow y(t) = 1 - 1.60078\, e^{-30t} \sin(20t)$

b) $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \dfrac{5(s+100)}{(s+10)(s+50)} = \dfrac{500}{500} = 1$

Solution

a) $\Rightarrow y(t) = u(t) - 1.125\, e^{-10t}\, u(t) + 0.125\, e^{-50t}\, u(t)$

2. 25 Pts. A four-wheel antilock automobile braking system uses electronic feedback to contr automatically the brake force on each wheel. A block diagram model for a brake control system is shown in figure P2, where $F_f(s)$ and $F_R(s)$ are the braking force of the front and r wheels, respectively, and $R(s)$ is tee desired automobile response on an icy road. Find $F_f(s)/R(s)$.

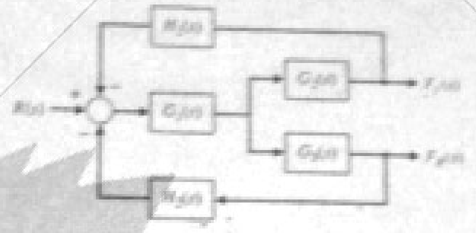
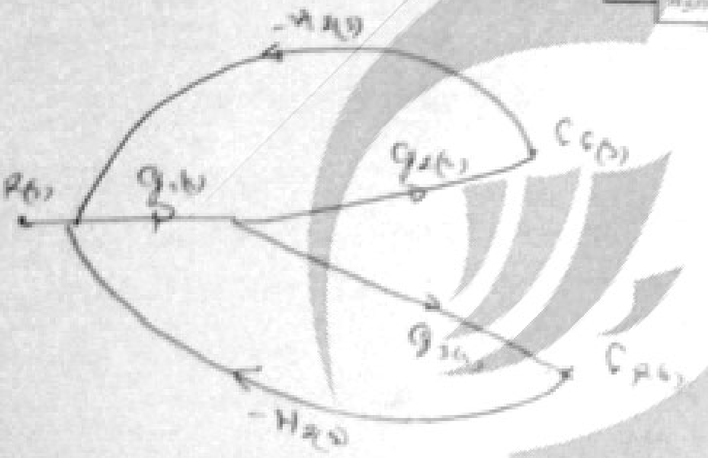


Figure P2.



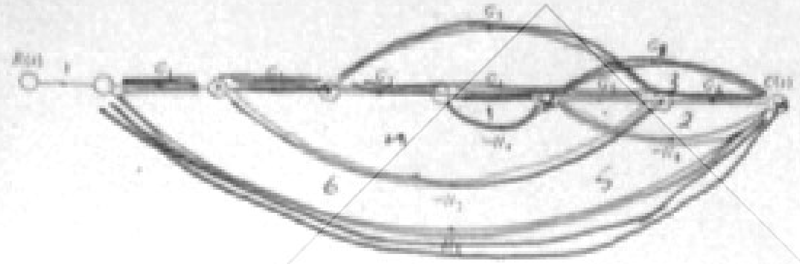
Path 1 $\Rightarrow M_1 = G_1 G_2$ $\qquad \Delta_1 = 1$

$$\Delta = 1 + G_1 G_2 H_2 + G_1 G_3 H_2 = 1 + G_1 H_2 (G_2 + G_3)$$

$$\Rightarrow \frac{F_{f(s)}}{R(s)} = \frac{M_1 \Delta_1}{\Delta} = \frac{G_1 G_2}{1 + G_1 H_2 (G_2 + G_3)}$$

(7)

5. 45 Pts. For the following signal-flow-graph, find C(s)/R(s). (Hint: there are 8 loops)



Path 1 = $G_1 G_2 G_3 G_4 G_5 G_6$; $\Delta_1 = 1$

Path 2 = $G_1 G_2 G_7 G_6$; $\Delta_2 = 1 - L_5 = 1 + G_4 H_4$

Path 3 = $G_1 G_2 G_3 G_4 G_8$; $\Delta_3 = 1$

Path 1: $G_1 G_2 G_3 G_4 G_5 G_6$
Path 2: $G_1 G_2 G_7 G_6$
Path 3: $G_1 G_2 G_3 G_4 G_8$

The Loops are,

$L_1 = -G_2 G_3 G_4 G_5 H_2$ $-G_2 G_3 G_4 G_5 H_2$

$L_2 = -G_5 G_6 H_1$ $-G_5 G_6 H_1$

$L_3 = -G_7 H_1$ $-G_7 H_1$

$L_4 = -G_2 H_2 G_7$ $-G_2 G_7 H_2$

$L_5 = -G_4 H_4$ $-G_4 H_4$

$L_6 = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$ $-G_1 G_2 G_3 G_4 G_5 G_6 H_3$

$L_7 = -G_1 G_2 G_7 G_6 H_3$ $-G_1 G_2 G_7 G_6 H_3$

$L_8 = -G_1 G_2 G_3 G_4 G_8 H_3$ $-G_1 G_2 G_3 G_4 G_8 H_3$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5 L_2 + L_5 L_4 + L_3 L_4)$$

2) $$\frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$$= \frac{(G_1 G_2 G_3 G_4 G_5 G_6) + (G_1 G_2 G_7 G_6)(1 + G_4 H_4) + (G_1 G_2 G_3 G_4 G_8)}{\Delta}$$